Structures of Rotating Traditional Neutron Stars and Hyperon Stars in the Relativistic $\sigma-\omega$ Model

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The influence of the rotation on the total masses and radii of the neutron stars are calculated by the Hartle's slow rotation formalism, while the equation of state is considered in a relativistic $\sigma-\omega$ model. Comparing with the observation, the calculating result shows that the double neutron star binaries are more like hyperon stars and the neutron stars of X-ray binaries are more like traditional neutron stars. As the changes of the mass and radius to a real neutron star caused by the rotation are very small comparing with the total mass and radius, one can see that Hartle's approximate method is rational to deal with the rotating neutron stars. If three property values: mass, radius and period are observed to the same neutron star, then the EOS of this neutron star could be decided entirely.

PACS: 04.40.Dg 95.30.Sf 97.10.Kc 97.60.Jd

I. INTRODUCTION

Neutron stars are dense, neutron-packed remnants of massive stars after their supernova explosions. Recently, in both experiment and theory, much concern is focused on the neutron stars [1–3]. One reason is to determine the equation of state (EOS) of superdense matter, and then understand the early universe, its evolution to the present day and the various astrophysical phenomena; the other one is that neutron star is one of the more possible sources of detectable gravitational waves. To understand neutron stars, the first thing is to understand its structure, such as the compositions, total masses, radii, redshifts etc. Because of the strongly gravitational field and the high density, neutron

stars must be studied in the framework of general relativity. For a static neutron star, giving EOS, using TOV equations [4] , which are educed from Einstein field equations, one can get an exact solution. But for a rotating neutron star, the components of Einstein field equations become much difficult. Nowadays, several approximate solutions of this problem were developed [5,6], in which Hartle's slow rotation formalism [7,8] is the most popular one.

The properties of neutron stars such as masses, rotational frequencies, radii, moments of inertia and redshifts are sensitive to the EOS of the matters [2,3]. As the interior core contains most of the mass of neutron star, so attention to the EOS is mostly focused on the core of neutron star, that is to the matters at density above several times of nuclear matter saturation density. In the core of neutron star, the compositions still keep blurry in some degree due to the high density and uncertainty of strong interaction, but as density increasing in the neutron star, neutrons can drip out of nuclei and form a neutron gas; and due to the URCA process $n \to p + e^- + \bar{\nu}_e$ and the neutrality of neutron star, there are at least protons and electrons in the chemical equilibrium; on the other hand, as the density increasing, hyperons will be dominant in the neutron stars [9]. In this letter, the Hartle's formalism will be used to deal with two kinds of rotating neutron stars: the traditional neutron stars, in which n, p, e, μ, are the main elements; and hyperon stars, in which $n, p, e, \mu, \Lambda, \Sigma, \Xi, \Delta$ are the main elements. The EOSs of them will be considered in the relativistic $\sigma - \omega$ model.

In this letter, we adopt the metric signature - + + +, G=c=1 (the factors G and c will be reinserted correctly in numerical calculation).

II. HARTLE'S SLOW ROTATION FORMALISM

In relativity, the space-time geometry of a rotating star in equilibrium is described by a stationary and axisymmetric metric of the form

$$ds^2 = -e^{2\nu}dt^2 + e^{2\lambda}dr^2 + e^{2\psi}(d\phi - \omega dt)^2 + e^{2\mu}d\theta^2 \tag{1}$$

where $\omega(r)$ is the angular velocity of the local inertial frame and is proportional to the star's rotational frequency Ω, which is the uniform angular velocity of the star relative to an observer at infinity. Expanding the metric function through second order in Ω, one has [7]

$$e^{2\nu} = e^{2\varphi}[1 + 2(h_0 + h_2 P_2)], \tag{2}$$

$$e^{2\lambda}=[1+\frac{2}{r}(m_0+m_2P_2)(1-\frac{2M_0(r)}{r})^{-1}](1-\frac{2M_0(r)}{r})^{-1}, \qquad (3)$$

$$e^{2\psi}=r^2sin^2\theta[1+2(v_2-h_2)P_2], \qquad (4)$$

$$e^{2\mu}=r^2[1+2(v_2-h_2)P_2], \qquad (5)$$

where $e^{2\varphi}$ and $M_0(r)$ denote the metric function and the mass of the nonrotaiting neutron star with the same central density, respectively; P_2 is the Legendre ploynomial of order 2; the perturbation functions m_0, m_2, h_0, h_2, v_2 are proportional to Ω^2 and are to be calculated from Einstein field equations.

From the (t,ϕ) component of Einstein field equations, one gets [7]

$$\frac{1}{r^4}\frac{d}{dr}(r^4j\frac{d\bar{\omega}}{dr})+\frac{4}{r}\frac{dj}{dr}\bar{\omega}=0, \qquad (6)$$

where$j(r)=e^{-\varphi}[1-2M_0(r)/r]^{\frac{1}{2}}$, $\bar{\omega}=\Omega-\omega$, which denotes the angular velocity of the fluid relative to the local inertial frame. The boundary conditions are imposed as $\bar{\omega}=\bar{\omega}_c$ at the center, $\frac{d\bar{\omega}}{dr}|_{\bar{\omega}_c=0}$, where $\bar{\omega}_c$ is chosen arbitrarily. Integrating eq.(6) outward from the center of the star, one would get the function $\bar{\omega}(r)$. Outside the star, from eq.(6) one has $\bar{\omega}(r)=\Omega-\frac{2J}{r^3}$, where J is the total angular mementum of the star, which takes the form [2] $J=\frac{1}{6}R_0^4\frac{d\bar{\omega}}{dr}|_{r=R_0}$. So at the surface, one can determine the angular velocity Ω corresponding to $\bar{\omega}_c$ as

$$\Omega=\bar{\omega}(R_0)+2\frac{J}{R_0^3}. \qquad (7)$$

From the (t,t) and (r,r) components of Einstein field equations, one gets two coupled ordinary differential equations of m_0 and h_0 as [7,8]

$$\frac{dm_0}{dr}=4\pi r^2\frac{d(p+\rho)}{dp}(\rho+p)p_0^*+\frac{1}{12}j^2r^4(\frac{d\bar{\omega}}{dr})^2-\frac{1}{3}r^3\frac{d(j^2)}{dr}\bar{\omega}^2, \qquad (8)$$

$$\frac{dp_0^*}{dr}=-\frac{m_0(1+8\pi r^2p)}{[r-2M_0(r)]^2}-\frac{4\pi r^2(p+\rho)}{r-2M_0(r)}p_0^*+\frac{1}{12}\frac{r^4j^2}{r-2M_0(r)}(\frac{d\bar{\omega}}{dr})^2+\frac{1}{3}\frac{d}{dr}[\frac{r^3j^2\bar{\omega}^2}{r-2M_0(r)}], \qquad (9)$$

where $p_0^*=-h_0+\frac{1}{3}r^2e^{-2v}\bar{\omega}^2+C$, here C is a constant determined by the demand that h_0 be continuous across the star's surface. These equations are also integrated outward, with boundary conditions that both m_0 and p_0^* vanish at the origin. With the same central density, the difference between the mass of the rotating star and the non-rotating star is

$$\delta M = m_0(R_0) + \frac{J^2}{R_0^3}, \tag{10}$$

The difference of the mean radius is

$$\delta r = -p_0^*(\rho + p)\frac{dp}{dr}. \tag{11}$$

III. MODEL FOR THE EOSS – THE RELATIVISTIC $\sigma - \omega$ MODEL

There are several models to deal with the superdense matters, such as non-relativistic models, relativistic fieldtheoretical models [3]. To different models, the fracions of particles in the superdense matters are different, and then the bulk properties of superdense matters are different, that is, the EOS of them are different. Here the Relativistic $\sigma - \omega$ Model will be adopted [10]. The Lagrangian density of this model is

$$\begin{aligned} L = \sum_B \bar{\psi}_B(i\gamma_\mu\partial^\mu + m_B - g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu - \frac{1}{2}g_{\rho B}\gamma_\mu\vec{\tau}.\vec{\rho}^\mu)\psi_B + \\ \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - U(\sigma) - \\ \frac{1}{4}\vec{\rho}_{\mu\nu}.\vec{\rho}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu.\vec{\rho}^\mu + \sum_l \bar{\psi}_l(i\gamma_\mu\partial^\mu - m_l)\psi_l \end{aligned} \tag{12}$$

in which $U(\sigma) = a\sigma + \frac{1}{3!}c\sigma^3 + \frac{1}{4!}d\sigma^4$, $F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$, $\vec{\rho}_{\mu\nu} = \partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu$, ψ_B is the field operator of Baryon B(B=n, p for the traditional neutron stars, B=n, p, Λ, Σ, Ξ, Δ for the hyperon stars); ψ_l is the field operator of lepton l(l=e, μ); and σ, ω^μ, $\vec{\rho}^\mu$ are the field operators of σ, ω, ρ meson respectively; $g_{\sigma B}$, $g_{\omega B}$, $g_{\rho B}$ are the coupling constants between σ, ω, ρ meson and baryon B respectively. In general, the coupling constants between σ(ω or ρ) meson and neutron and proton are equal and decided by saturated property of nuclear matter($g_{\sigma n} = g_{\sigma p}, g_{\omega n} = g_{\omega p}$) and the symmetry energy of nuclear matter($g_{\sigma\rho}, g_{\omega\rho}$); $m_B, m_l, m_i, (i = \sigma, \omega, \rho)$ are the mass of baryon, lepton, meson respectively; $\vec{\tau}$ is the isospin operator. To leptons, we assume they are free fermi gas. From this Laglangian density, we can obtain the EOS of the superdense matters, $p = p(\rho)$, in which, p and ρ are the pressure and energy density of the superdense matters, respectively. In 1-loop approximation, the loop's contribution to the propagators of nucleon and σ meson is considered, and the renormalization is used to renormalizing the divergent part of loop contribution. In the numerical calculation, we adopt the value of the parameter as following [10]: $a = -2.1 \times 10^7 MeV^3$, $c = 0.97 \times M_n$, $d = 1277$, $g_s = 6.73$, $g_v = 8.59$, $M_n = 938MeV$, $m_\omega = 783MeV$, $m_\sigma = 550MeV$, $m_\rho = 770MeV$, and the incompressibility of

nuclear matter is $224MeV$, which is consistent with the experiment result [11,12]. From Fig.1, it is clear that the EOS of traditional neutron stars is stiffer than the EOS of hyperon stars.

Fig.1

IV. NUMERICAL RESULTS AND DISCUSSION

In order to compare the numerical results with the observation, we present some typical observation value here. As we know, only a few masses have been determined in observation from the more than thousand neutron stars. There are two typical observed mass data: $1.36 \pm 0.08 M_{\odot}$ [13] for double neutron star binaries, and $1.87^{+0.23}_{-0.17} M_{\odot}$(Vela X-1) [14] and $1.8 \pm 0.4 M_{\odot}$(Cygnus X-2) [15] for X-ray binaries.

The numerical results for the rotating traditional neutron stars and hyperon stars in the relativistic $\sigma - \omega$ model are showed in the following figures and table.

Figs.2-3 show the changes of masses and radii between rotating neutron stars and non-rotating neutron stars as function of rotational period. At a given central density, it is easy to see that, as the neutron stars rotating slowly, the increment of the masses and radii will reduce sharply when the neutron stars' rotational periods are small than 1 ms. But as the periods become bigger than 1.6ms, which is the period of the fastest rotating pulsars in observation [3], the change of the increment of the masses and radii caused by the rotation will become very week, and the increment will be no more than 2 percent. As the changes of the mass and radius to a real neutron star caused by the rotation are very small comparing with the total mass and radius, we can see that Hartle's approximate method is rational.

Figs.4-7 show the masses and radii of rotating and non-rotating traditional neutron stars and hyperon stars as function of central density at a given central angular velocity relative to the local inertial frame. From these figures, one can see that around the typical observational radii with value of 12km, the increments of the radii are bigger than that of other place. In table 1, some typical value in these figures are listed, combining fig.4, fig.6 and the observational value, one can see that the double neutron star binaries are more like hyperon stars, while the neutron stars of X-ray binaries are more like traditional neutron stars. For the hyperon stars, its EOS is so soft that, even the period small than the smallest observational value, the total mass can't reach the observation value of X-ray binaries, about $1.8 M_{\odot}$.

Fig.8 shows the period as a function of the central density at two giving central angular velocities relative to the local inertial frame. From this figure, one can see that bigger central density and bigger central angular velocity relative to the local inertial frame correspond with a smaller period. In fig.8 one can find out that as the EOS of hyperon star is softer, at the same conditions, the period of hyperon star is appreciably bigger than the period of the traditional neutron star. Figs.9-10 show the masses and radii of the traditional neutron stars and hyperon stars as a function of period. One can see that even the central density of the hyperon stars is bigger than that of the traditional neutron stars, the masses and radii of the hyperon stars are smaller than that of the traditional neutron stars. Another interesting result is that at a giving central density, when the period increases to a value which is bigger than the smallest observational period, 1.6ms, the change of the masses and the radii with the period is not obvious, which is consistent with the result of fig.2.

From these figures, one can see that at a giving period, the central density and the central angular velocity relative to the local inertial frame could be chosen freely. but if the masses and the period of a neutron star are given (by the observational value) at the same time, then the central density and the central angular velocity relative to the local inertial frame will be decided. As we know, there is another observational value: the radius of the neutron star, the decided central density and central angular are not always given the exactly radius. So one can say that if these three observational values: mass, radius and period are given to the same neutron star, there is only one special EOS could give a set of calculating values, which are fit like a glove to the set of observational value, that is, these three observational values could decide the EOS entirely. But the problem is that there is no neutron star with both mass and radius observationally determined up to the present.

Fig. 2-Fig. 10

Table 1. Rotating neutron star's property in the relativistic $\sigma - \omega$ model*

	$\bar{\omega}_c$	ρ_c	$R_0(km)$	M_0	$R(km)$	M	$\frac{\delta R}{R_0}$	$\frac{\delta M}{M_0}$	$P(ms)$
TNS	2.50	1.067	11.98	1.768	12.18	1.808	0.017	0.023	1.488
	5.00	0.903	10.21	1.650	12.94	1.801	0.070	0.092	0.791
HS	2.50	0.921	11.80	1.336	12.04	1.367	0.020	0.023	1.686
	5.00	0.811	10.87	1.265	12.87	1.385	0.084	0.095	0.873

*In the table, TNS and HS denote the traditional neutron stars and the hyperon stars respectively; $\bar{\omega}_c$ is the angular velocity relative to the local inertial frame at the center, with a unit of $(10^3 s^{-1})$; ρ_c is the central density, with a unit of $(10^{18} kg.m^{-3})$; R_0 and M_0 denote the radius and the mass of the non-rotating stars, R and M denote the radius and the mass of the rotating neutron stars, respectively; the unit of the mass is the solar mass($M_\odot$), $\frac{\delta R}{R_0}$ and $\frac{\delta M}{M_0}$ are the fractional difference of the radius and mass between the rotating neutron stars and non-rotating neutron stars; P is the rotational period.

Acknowledgements

The project supported by National Natural Science Foundation of China (Grant No. of 10275099, 10175096), GuangDong Provincial Natural Science Foundation (Grant No. of 021707) and Natural Science Foundation of SCUT (Grant No. of E5123266).

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V. FIGURE CAPTIONS

Fig. 1. Equation of state of traditional neutron stars(TNS) and hyperon stars(HS).

Fig. 2. Increment of the mass and radius of TNS comparing to that of non-rotating TNS at the center density of $\rho_c = 3\rho_0$, where ρ_0 is the saturation density of nuclear matters.

Fig. 3. Increment of the mass and radius of HS comparing to that of non-rotating HS at the center density of $\rho_c = 4\rho_0$.

Fig. 4. The total mass of the non-rotating and rotating TNS as a function of central density, where the mass is in a unit of solar masses.

Fig. 5. The radius of the non-rotating and rotating TNS as a function of central density.

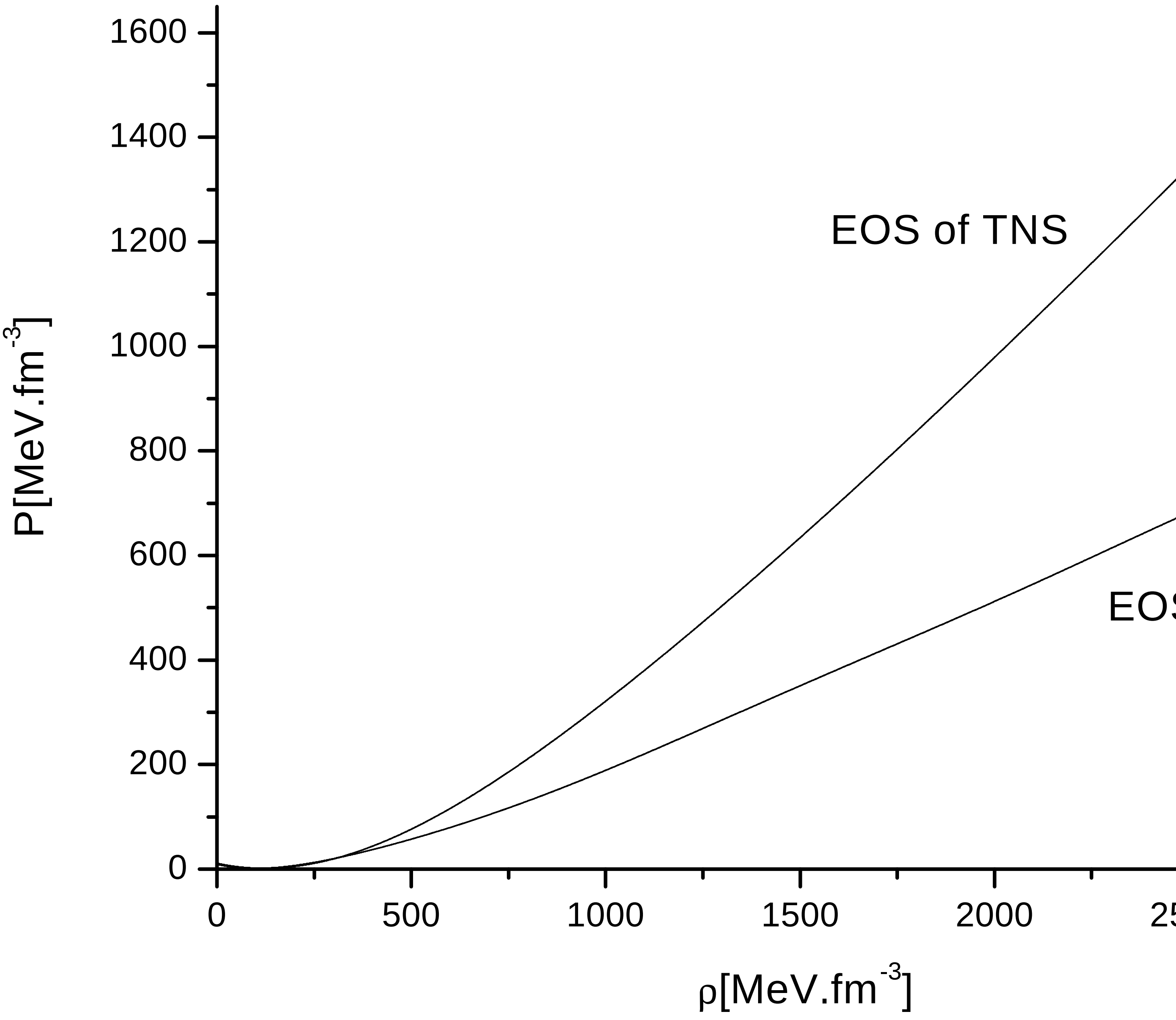
Fig. 6. The total mass of the non-rotating and rotating HS as a function of central density, where the mass is in a unit of solar masses

Fig. 7. The radius of the non-rotating and rotating HS as a function of central density.

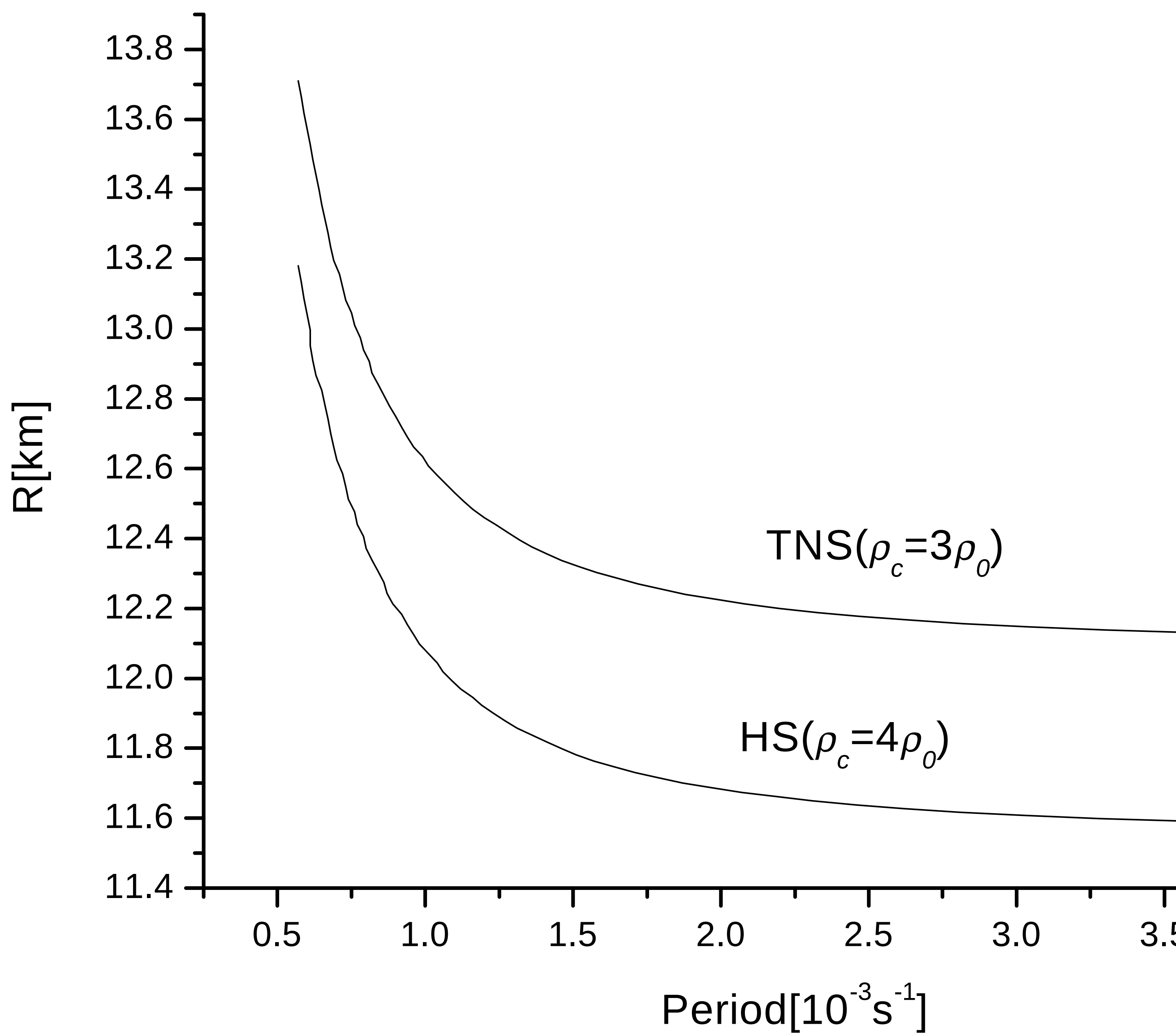
Fig. 8. Period as a function of the central density at two different angular velocities relative to the local inertial frame at the center.

Fig. 9. The masses as a function of the period at two different central density for the TNS and HS.

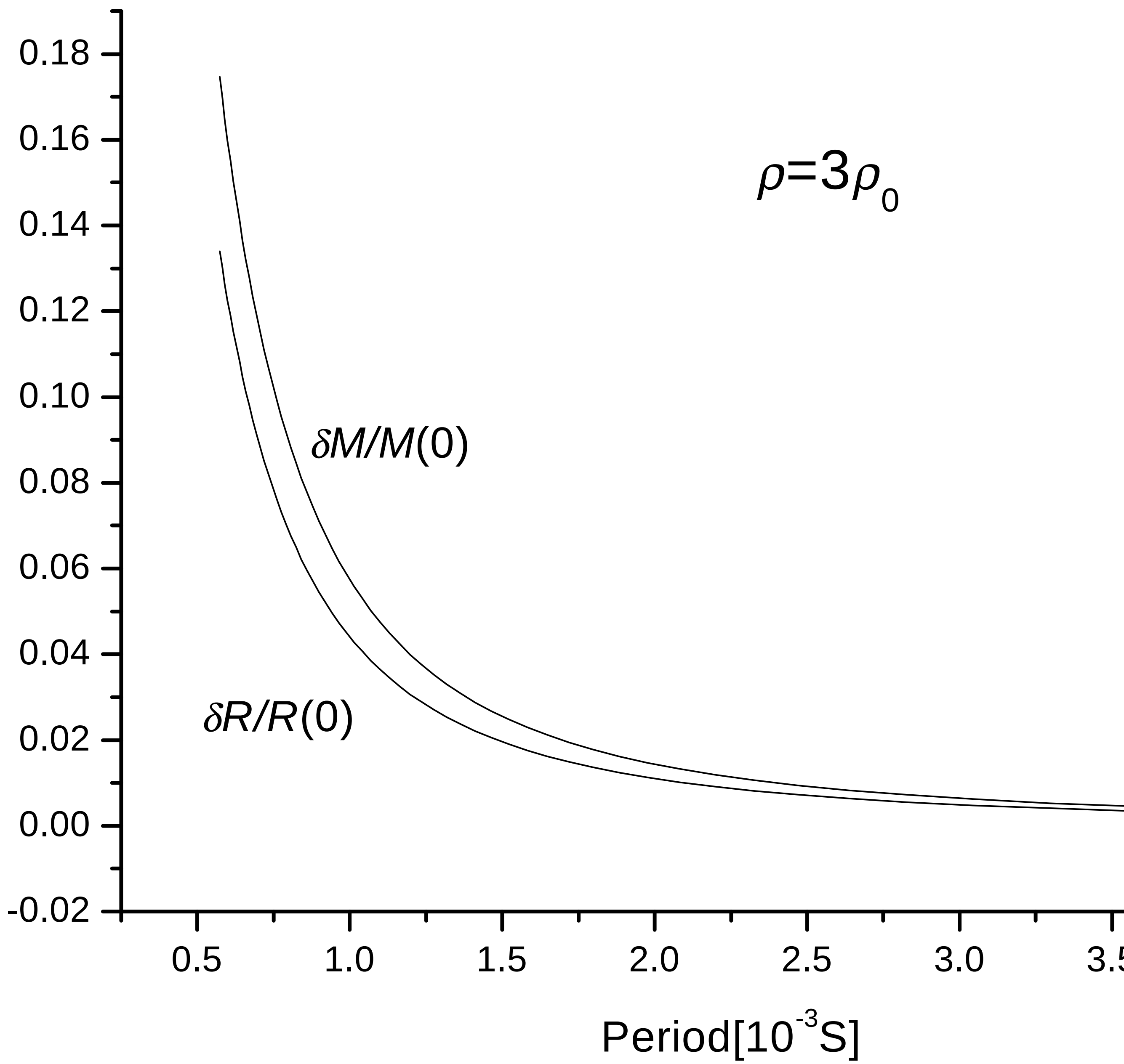
Fig. 10. The radii as a function of the period at two different central density for the TNS and HS.



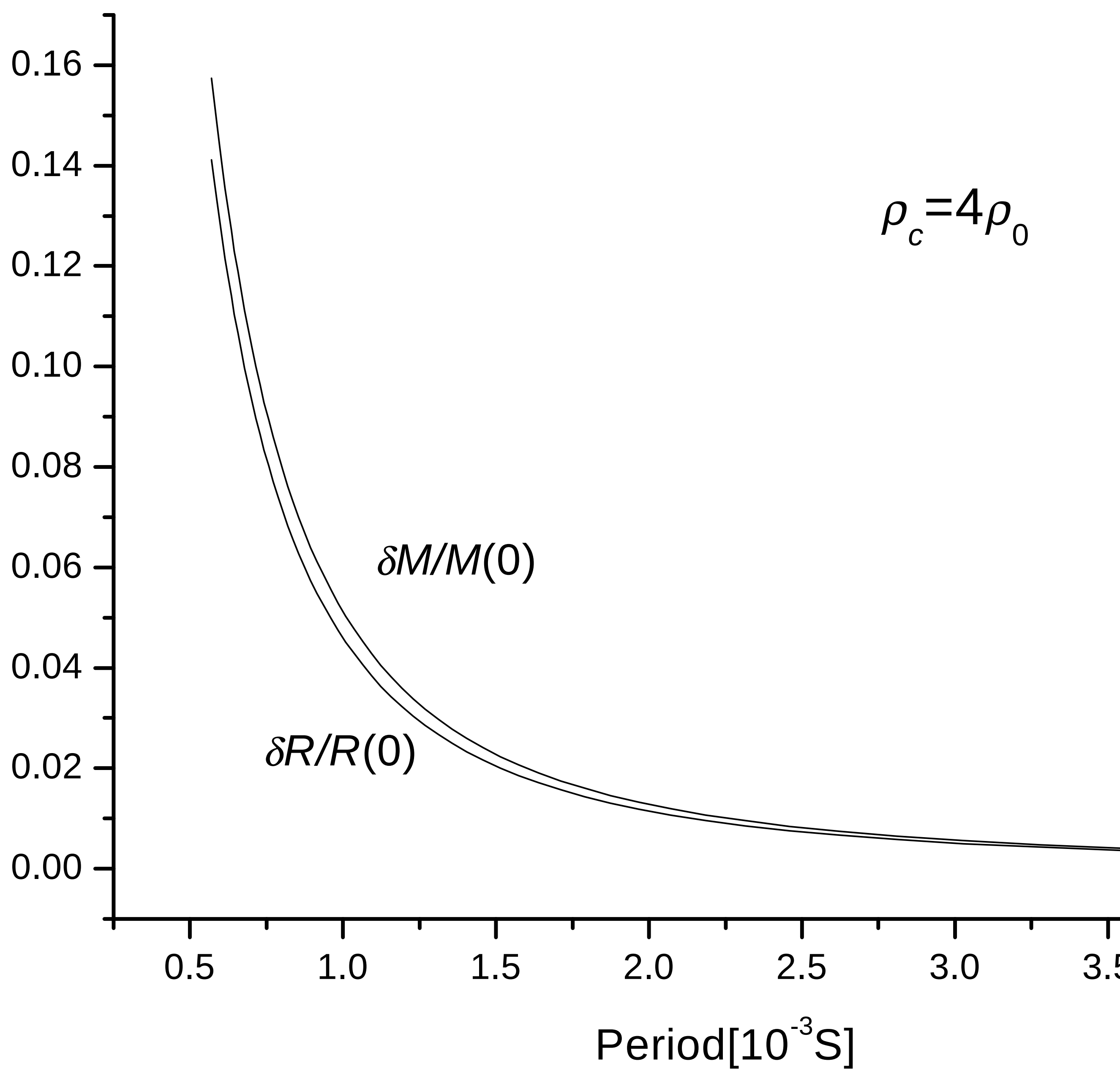
1600
1400
1200
1000
800
600
400
200
0
0
500
1000
1500
2000
P[MeV.fm^{-3}]
ρ[MeV.fm^{-3}]
EOS of TNS

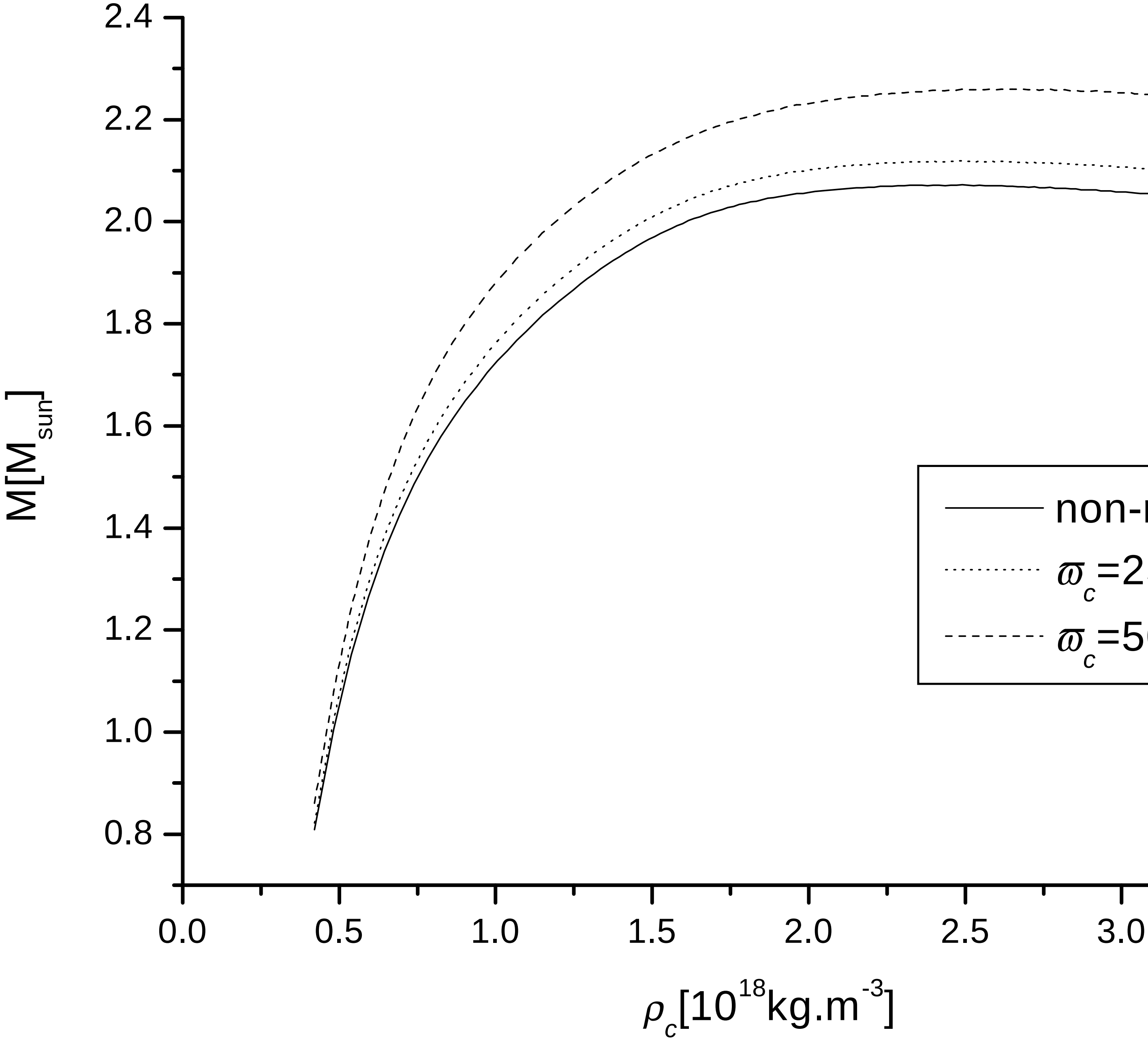


R[km]
13.8
13.6
13.4
13.2
13.0
12.8
12.6
12.4
12.2
12.0
11.8
11.6
11.4
0.5
1.0
1.5
2.0
2.5
3.0
3.5
Period[10^-3s^-1]
TNS(ρc=3ρ0)
HS(ρc=4ρ0)

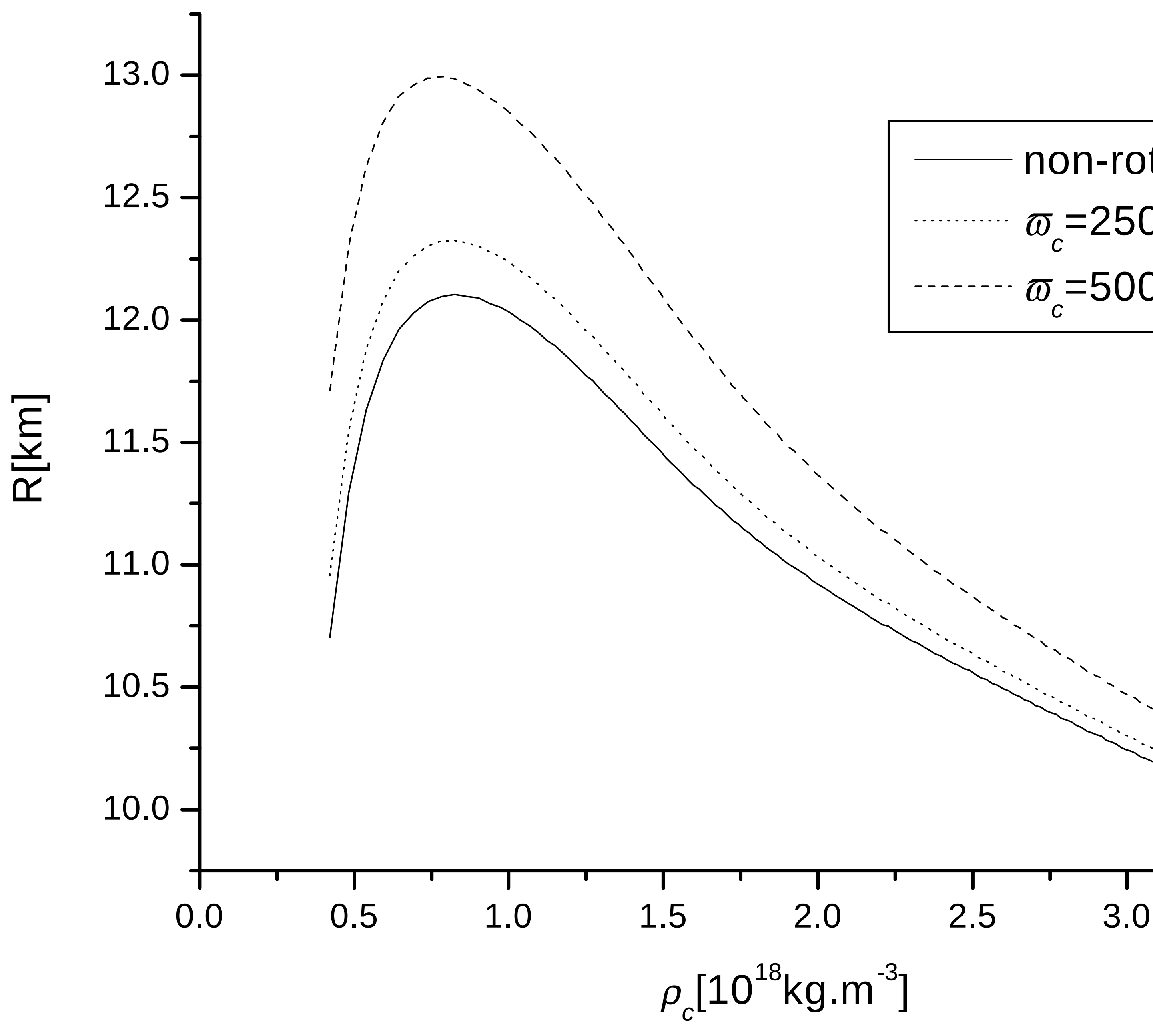


0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
-0.02
0.5
1.0
1.5
2.0
2.5
3.0
3.5
$\rho=3\rho_0$
$\delta M/M(0)$
$\delta R/R(0)$
Period[10^{-3}S]

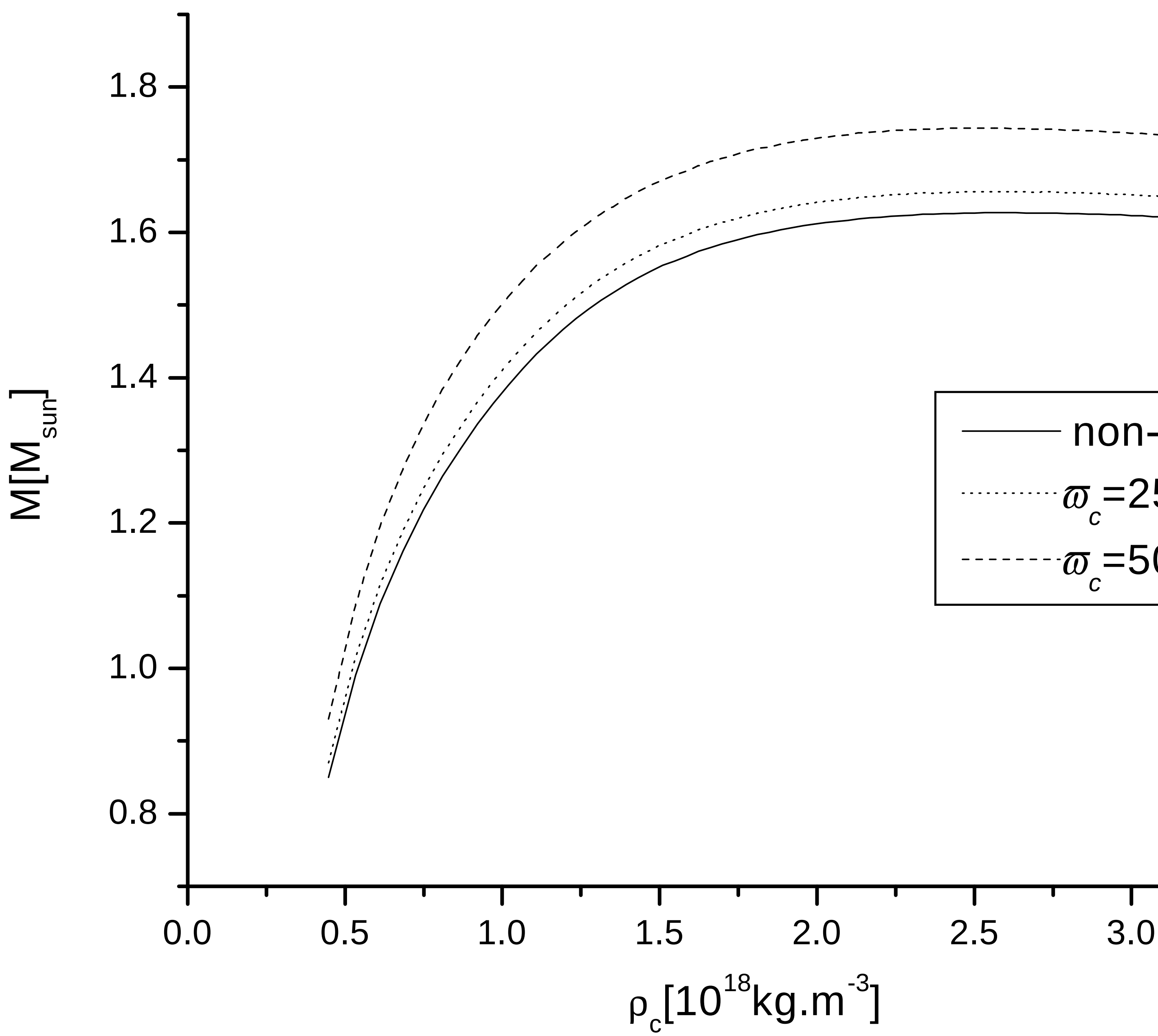

0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
$\rho_c=4\rho_0$
$\delta M/M(0)$
$\delta R/R(0)$
0.5
1.0
1.5
2.0
2.5
3.0
Period[10^{-3}S]



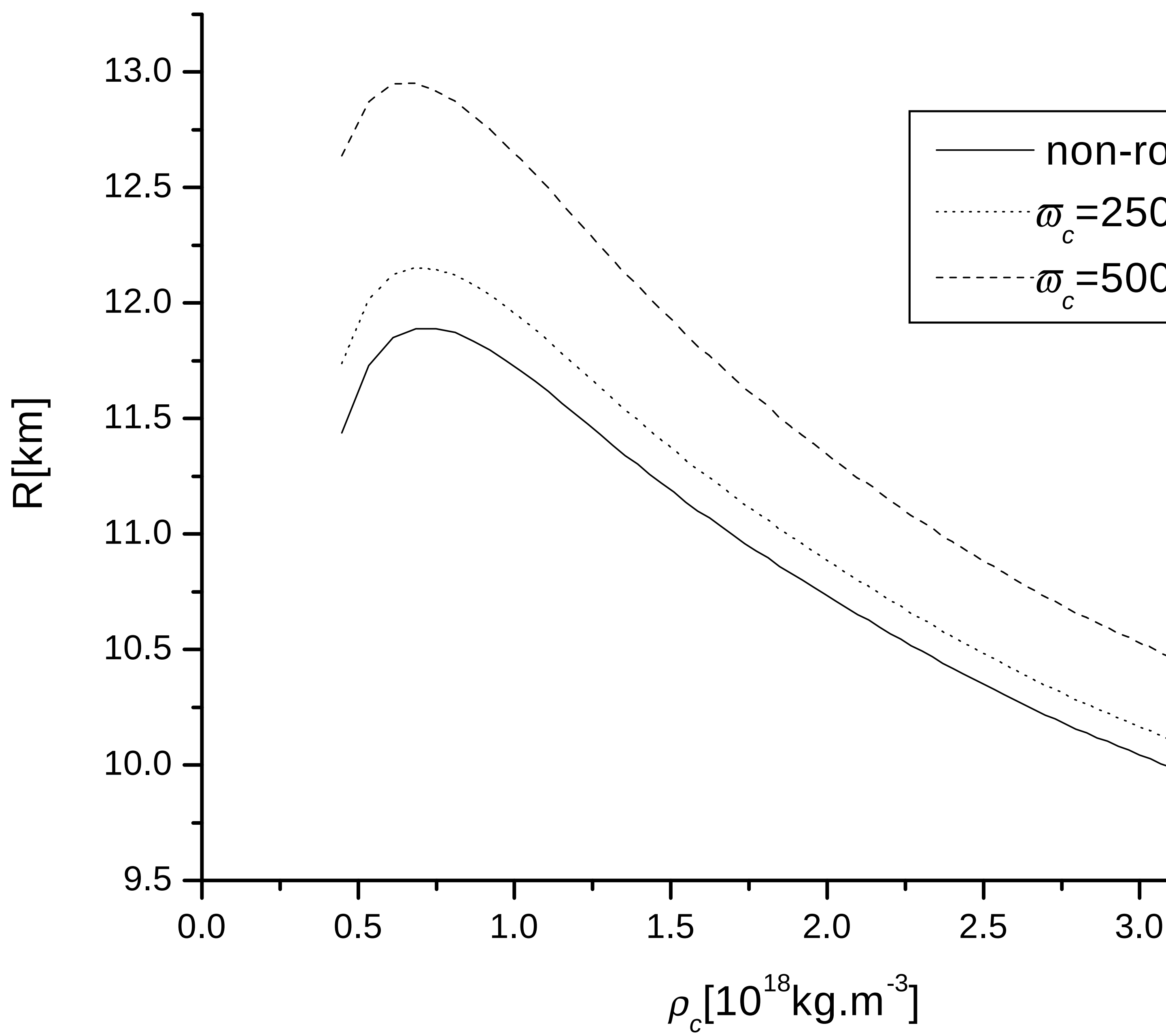
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
M[M_sun]
0.0
0.5
1.0
1.5
2.0
2.5
ρ_c[10^18 kg.m^-3]
non-r
ϖ_c=2
ϖ_c=5



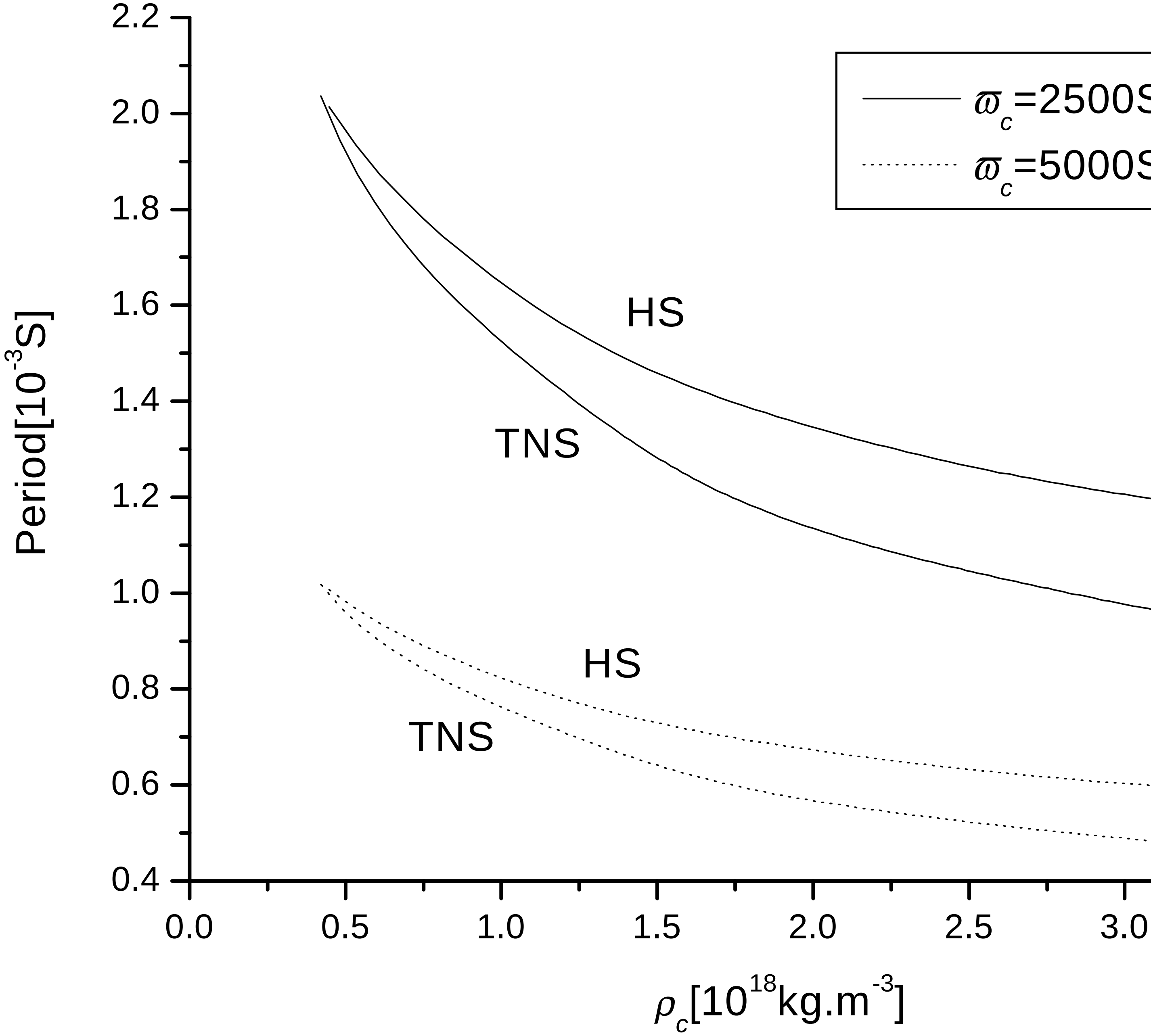
13.0
12.5
12.0
11.5
11.0
10.5
10.0
R[km]
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$\rho_c[10^{18}kg.m^{-3}]$
non-rot
ϖ_c=250
ϖ_c=500



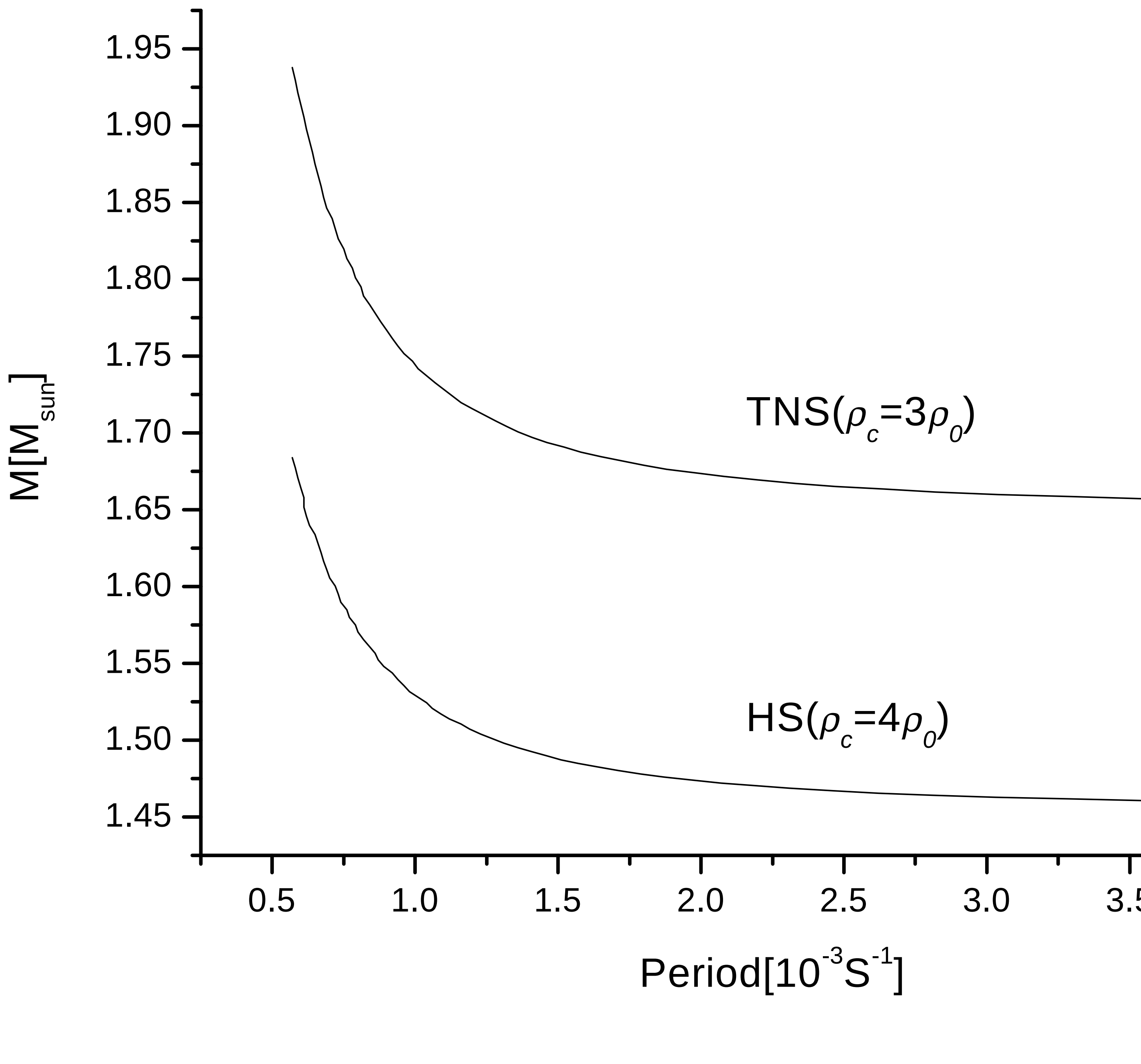
1.8
1.6
1.4
1.2
1.0
0.8
M[M_sun]
0.0
0.5
1.0
1.5
2.0
2.5
3.0
ρc[10^18 kg.m^-3]
non-
ϖc=25
ϖc=50



R[km]
13.0
12.5
12.0
11.5
11.0
10.5
10.0
9.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$\rho_c[10^{18}\mathrm{kg.m}^{-3}]$
non-ro
ϖ_c=250
ϖ_c=500



2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
Period[10^-3S]
0.0
0.5
1.0
1.5
2.0
2.5
3.0
ρc[10^18kg.m^-3]
ϖc=2500S
ϖc=5000S
HS
TNS
HS
TNS



M[M$_{sun}$]
1.95
1.90
1.85
1.80
1.75
1.70
1.65
1.60
1.55
1.50
1.45
TNS(ρ_c=3ρ_0)
HS(ρ_c=4ρ_0)
0.5
1.0
1.5
2.0
2.5
3.0
Period[10^{-3}S^{-1}]